# Statistical inference for poverty lines and measures: Theory and application

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**Abstract**

The measurement of monetary poverty is fundamentally based on the identification of a poverty line that distinguishes the poor from the non-poor. Although this threshold plays a central role in the analysis and comparisons of poverty levels over time and space, it is typically assumed to be known with certainty. This is for example the case with the concave poverty lines (CPL) developed by ESCWA, which facilitate international comparisons to be made. However, such an assumption overlooks the statistical uncertainty inherent in the estimation of poverty lines. In this article, we propose methods for approaching the uncertainty into both predicted poverty lines and the poverty measures derived from them. We develop four methods to address this by constructing confidence intervals for poverty lines and FGT poverty indexes. These methods are based on interval projection, optimization and/or bootstrap techniques. Using the international database developed by ESCWA and the estimated CPL model, we apply these methods to the case of Morocco, using five national households surveys conducted between 2001 and 2019. The results show that the uncertainty about poverty lines is not negligible, although its impact on FGT indices remains limited in a context of low poverty. This article makes an original contribution to enhancing the robustness of poverty assessments and addresses growing concerns regarding the international comparisons of poverty measures.

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## 1 Introduction and motivation

Basically, there are several approaches to measuring monetary (income or expenditure) poverty. When it comes to measuring poverty at the country level, whether at the national level or by area or region, we find two different types of poverty lines, the absolute and the relative. Other approaches, developed by the World Bank, exist to allow comparisons over time and space by setting subjectively a threshold expressed in US dollars at purchasing power parity (PPP). However, there seems to be no consensus on a threshold for measuring poverty in a country that is also comparable worldwide.

Furthermore, researchers sometimes face limited or no access to raw survey data to set poverty thresholds and measure its incidence and scope. Several authors concerned with measuring poverty in the absence of access to surveys have developed methods to circumvent this problem. For example, Datt (1998) deals with this particular problem by using data grouped into deciles. He proposes statistical tools based on parameterization of Lorenz curves and their numerical translations to estimate measures of poverty and inequality. Groß et al. (2017), Walter and Weimer (2018) and Walter (2019) have introduced a technique known as data augmentation. This method takes into account the fact that data are available for the variable of interest, but only in intervals and not for individual data. The estimated density must then respect and reproduce the consistent point estimates that are already available. Makdissi et al. (2022) also propose a method for using partial, accessible data to extract maximum information on poverty and inequality. In their approach, they estimates distribution functions from interval data. They infer the limits of the cumulative distribution, taking into account the points for which estimates are available. This method is then used to measure poverty for given thresholds and to make stochastic dominance comparisons (in terms of poverty).

However, to estimate the measure of poverty, all these authors need a given poverty line, even though they are faced with great uncertainty. In ESCWA (2022), the authors propose a new approach to estimating national poverty lines that are comparable between countries and over time. This method, called *concave poverty line* (CPL), is based on the principle of a concave relationship between the level of income or total expenditure and the poverty line.

With this method, the Economic and Social Commission for Western Asia (ESCWA) makes it possible to estimate internationally comparable poverty lines and to deduce punctual poverty measures. No approach is taken to the uncertainty associated with this estimation of thresholds, and comparisons over time and space lack robustness. In this article, we precisely approach the problem of uncertainty from two axes, which is broken down into four distinct methods. The first, which we describe as classical, involves constructing confidence intervals for poverty lines and measures based on projection techniques. Two methods have been developed to construct confidence intervals for poverty lines. One method is based on a criterion for optimizing the range of the interval (Method A), and the other uses the Delta method to construct the confidence interval (Method B). Due to the limitations of these methods, the second axis presents an alternative approach based on the bootstrap. Two new methods are used to construct confidence intervals. The first (Method C) involves bootstrapping a confidence interval for the poverty line to obtain a confidence interval for the FGT measures. The second (Method D) relies entirely on the bootstrap approach to derive confidence intervals for these poverty measures. We illustrate these methods by combining the ESCWA CPL approach with Moroccan survey data. We present and discuss comparisons of the results obtained from these four methods.

In the next section, we will briefly review the concepts of poverty lines and measures (Section 2). We present the concave poverty line approach developed by ESCWA (2022), to which we apply the Duan (1983) method to correct the bias associated with the exponential transformation. The classical approach and the two methods derived from it are presented in Section 3. Section 4 introduces the bootstrap approach to constructing confidence intervals for the line or measures of poverty. After presenting the Moroccan data used for illustration in Section 6, we analyze and compare the results obtained. A conclusion is provided in the Section 7.

# 2 On the importance of poverty lines in measuring monetary poverty

## 2.1 About the poverty lines

As previously mentioned, two main approaches are generally used to identify these poverty lines: absolute and relative thresholds. These approaches are complemented by an international poverty line proposed by the World Bank and the

ESCWA to allow for comparisons over time and space.

The absolute poverty line is defined as the level of consumption or income required to satisfy a minimum basket of basic needs, including food needs (reference caloric intake) and basic non-food needs. This method is often based on the cost of basic needs method formalized by Ravallion (1998). It has been widely used in developing countries. However, despite its contextual relevance, this approach fails to reflect changing living standards. A line set at one time may become obsolete if it is not revised. What's more, the non-food component of the threshold is based on sometimes subjective and locally dependent choices, which limits international comparability (Ravallion, 2008).

The relative poverty line is generally defined in terms of the distribution of living standards in a country (50% or 60% of median consumption or income). It is more widely used in high-income countries and aims to capture forms of social exclusion and inequality. This approach is highly dependent on the structure of the income distribution, which can lead to paradoxical interpretations. For example, forAtkinson et al. (2002), relative poverty captures inequalities more than material deprivation. This approach also does not allow for international comparisons. To this end, in the early 1990s the World Bank adopted an international poverty line based on a expenditure threshold expressed in U.S. dollars at purchasing power parity (PPP) that intended to reflect a comparable level of minimum well-being across countries. This threshold has evolved from 1 dollar/day, based on the poorest countries (Ravallion et al., 1991), to 1.90 $ PPP in 2015 (PPP 2011) (Ferreira et al., 2016). However, these thresholds are based on the national thresholds of the poorest countries, making them of little relevance to middle-income countries. Moreover, as Deaton (2010) points out, PPP estimates have to be revised periodically, making comparisons over time problematic. Given these limitations, Lanjouw and Ravallion (1995) argue for poverty lines that are locally anchored but internationally comparable.[1][2]

Due to the lack of consensus on methods for establishing poverty lines, especially for international comparisons, the ESCWA has proposed a new approach to estimating these key parameters in income poverty measurement.

[1] See Ravallion (2012) for a detailed presentation of poverty line construction approaches.
[2] There are also poverty lines based on subjective criteria (Ravallion (2012)).

## 2.2 On the concave poverty line of ESCWA

In ESCWA (2022), the authors propose a new approach to estimating national poverty lines that are comparable across countries and over time. This method, called the *Concave Poverty Line* (CPL), is based on the concave relationship between the poverty line and mean of the income or total expenditure levels. It considers that the cost of basic needs increases with income or expenditure but at a decreasing rate. Unlike linear poverty lines, such as the World Bank's *Societal Poverty Line* (SPL), which grows proportionally to median expenditure, the CPL establishes an upper asymptotic bound on the growth of the poverty line. This property reflects the fact that, in high-income countries, the cost of basic necessities rises less as average expenditure rises, in line with Engel's law that the share of expenditure devoted to basic necessities declines with expenditure.

The ESCWA's study has two objectives: first, to obtain poverty lines that reflect a country's level of development and economic realities; and second, to ensure the cross-country and cross-temporal comparability needed to monitor progress toward the Sustainable Development Goals (SDGs).

Formally, according to Engel's law, the relationship between the poverty line, denoted by $z_{i,t}$, and the mean expenditure *per capita*, denoted by $M_{i,t}$, of a country, designated by the index $i$, at a specific date, denoted by $t$, is assumed to be concave. This assumption contradicts the SPL linearity hypothesis.

ESCWA (2022) defined the CPL model by the following relationship:

$$\frac{z_{i,t}}{M_{i,t}} = \gamma \cdot (\log M_{i,t})^{-1} \tag{1}$$

with $z_{i,t}$, the poverty line expressed in 2011 PPP/day, $M_{i,t}$, the average income *per capita* of country $i$ at date $t$ and $\gamma$ a scaling parameter. After logarithmic transformation, the ESCWA proposes to estimate the following model[3]:

$$\log z_{i,t} = \alpha_0 + \beta_1 \log M_{i,t} + \beta_2 C_i + \varepsilon_{i,t} \tag{2}$$

where $C_i$ is a binary variable taking the value 1 for countries using expenditure (*vs.* income) data and $\varepsilon_{i,t}$, a random error term.[4] This specification allow to impose

[3]Assuming that $M_{i,t}$ is very large, we have $\log M_{i,t} \gg \log(\log M_{i,t})$ and consequently, to first order, we can write: $\log M_{i,t} - \log(\log M_{i,t}) \approx \log M_{i,t}$.

[4]The authors of ESCWA (2022) add a binary variable noted $D_i$ which takes the value 1 to identify countries with liberal economies. This variable was not included in the estimations,

an asymptotically bounded growth of poverty lines with income, unlike the SPL model.

After estimating the model (Equation 2), it can be used to predict the poverty line, $\hat{z}_{i,t}$, based on the data of the variables, $M_{i,t}$ and $C_i$. The results demonstrate that the CPL model respects the axiom of weak relativity, even with a limited number of covariates. The CPL model also provides better projections of poverty lines in wealthy countries than the SPL model.

The authors propose several robustness tests to validate the CPL specification. They also report that they corrected for residual skewness and inherent non-normality. However, the correction made in particular to recover the predicted poverty lines in level is not clearly spelled out in ESCWA (2023a). But Duan (1983) demonstrates the prediction bias that results from not taking this non-normality into account in *retransformation* and suggests a method for correcting it.

## 2.3 Feedback on Duan's correction

The log-linear model used in ESCWA (2022) can be generalized and written as follows $\ln(z_{i,t}) = x'_{i,t}\beta + \varepsilon_{i,t}$ to model the poverty line $z_{i,t}$, assumed to be strictly positive. In addition, it is assumed that the $\varepsilon_{i,t}$ errors are *i.i.d*, normally distributed with mean zero and variance $\sigma^2$. After estimation, it is possible to predict $z_{i,t}$ by applying the usual exponential transformation. However, Duan (1983) shows that transforming the prediction to obtain $\hat{z}_{i,t} = \exp(x'_{i,t}\hat{\beta})$, introduces a systematic bias in the estimation of $E[z_{i,t}|x_{i,t}]$, called "*transformation bias*". This bias is due to inappropriate use of the normality assumption. Failure to take this problem into account leads to an inference that Greene (2012) calls *naive*.

Formally, even assuming $E[\varepsilon_{i,t}|x_{i,t}] = 0$, the conditional expectation of $z_{i,t}$ is written $E[z_{i,t}|x_{i,t}] = E[\exp(x'_{i,t}\beta + \varepsilon_{i,t})|x_{i,t}] = \exp(x'_{i,t}\beta)E[\exp(\varepsilon_{i,t})|x_{i,t}]$. However, nothing guarantees that $E[\exp(\varepsilon_{i,t})|x_{i,t}] = 1$. It is shown that under the assumption of normality, $E[\exp(\varepsilon_{i,t})] = \exp(\sigma^2/2)$. Consequently, we obtain $\hat{z}_{i,t} = \exp(x'_{i,t}\hat{\beta} + \hat{\sigma}^2/2)$ with $\hat{\beta}$ and $\hat{\sigma}^2$ deduced from the estimated model.

To correct for this bias without postulating strong parametric assumptions, Duan (1983) proposes a non-parametric method called *smearing estimator* denoted $\hat{h}$. This approach consists in estimating $E[\exp(\varepsilon_{i,t})]$ from the empirical mean of the

which is precisely the reason we removed it from the Equation 2 .

exponential of the residuals of the estimated model such that:

$$\hat{h} := \frac{1}{n} \sum_{t=1}^{T} \sum_{i=1}^{n} \exp(e_{i,t}), \qquad \text{with } e_{i,t} = \ln(z_{i,t}) - x'_{i,t}\hat{\beta}, \tag{3}$$

and correct the punctual prediction by taking into account $\hat{h}$ such that:

$$\hat{z}_{i,t} = \hat{h} \cdot \exp(x'_{i,t}\hat{\beta}). \tag{4}$$

The application of Duan's method thus results in more robust predictions. However, this prediction of $\hat{z}_{i,t}$ (corrected or not) remains punctual and is generally used to calculate poverty measures.[5]

## 2.4 About the monetary poverty measures

To approach the levels of monetary poverty, we use the well known indices developed by Foster et al. (1984), noted $FGT_a$ which are written:[6]

$$FGT_a = \int_0^z \left( \frac{z - x}{z} \right)^a f(x)\, dx, \tag{5}$$

with $z$ the poverty line and $x$ the expenditure *per capita*, $f(x)$ the density function of $x$. By varying the value of the poverty aversion parameter ($a$), these measures generate the incidence of poverty ($a = 0$), its depth ($a = 1$) and the degree of its severity ($a = 2$). The latter measure further weights deviations from the poverty line, thereby capturing the inequality between the poor. It is trivial that for a given density function, these measures are monotonically increasing with $z$.

In general, the density $f(x)$ is high in the neighborhood of $z$. Consequently, even small variations in $z$ can result in significant variations in poverty measures. Furthermore, any uncertainty that is unavoidable when estimating the poverty line will affect poverty measures systematically. The article aims to address this uncertainty by constructing confidence intervals for both predicted lines and measures of poverty. We approach the problem of uncertainty from two axes, classical and bootstrap, which are broken down into four distinct methods.

[5] In the remainder of this article, we will omit the subscripts $i$ and $t$ to simplify notation.

[6] We write the poverty aversion parameter as $a$ instead of the $1 - \alpha$ usually used in the literature, since $\alpha$ will be adopted in this article to define the significance level in the construction of confidence intervals.

# 3 Confidence intervals by classical approach

A first intuitive approach, but considered as *naive* by Greene (2012), is to deduce a confidence interval for $z$ by directly projecting the confidence interval we would construct for $\ln z$ after estimating the Model 2. Formally, after estimating the model, we construct the following interval:

$$\begin{aligned} CI_A^{\ln z} = \left[\widehat{\ln z_{\mathrm{A}}^-}, \widehat{\ln z_{\mathrm{A}}^+}\right] = \Big[ & x'\hat{\beta} - t_{(1-\alpha/2),[nT-K]} \cdot se\left(e_z\right), \\ & x'\hat{\beta} + t_{(1-\alpha/2),[nT-K]} \cdot se\left(e_z\right) \Big]. \end{aligned} \tag{6}$$

with $\alpha$ given, $K$ the number of regressors in the estimated model and $se\left(e_z\right)$ the estimate of the standard deviation of the prediction associated with $\ln z$. By simple transformation, we can deduce:

$$\begin{aligned} CI_A^{zn} = \left[\hat{zn}_{\mathrm{A}}^-, \hat{zn}_{\mathrm{A}}^+\right] = \Big\{ & \exp\left[x'\hat{\beta} - t_{(1-\alpha/2),[nT-K]} \cdot se\left(e_z\right)\right], \\ & \exp\left[x'\hat{\beta} + t_{(1-\alpha/2),[nT-K]} \cdot se\left(e_z\right)\right] \Big\}. \end{aligned} \tag{7}$$

The *naivety* of this approach stems from the fact that $\ln z$ is assumed to be normal and therefore symmetrical, whereas $z$ is necessarily asymmetrical, as shown in Figure 2. Consequently, the resulting confidence interval cannot be optimal. Greene (2012) proposes a method for making this interval optimal.

## 3.1 Optimized confidence intervals

To construct a Greene-optimized confidence interval for the poverty line $z$, we consider a constrained optimization problem. From the previous naive formulation and by projection, we have:

$$\begin{aligned} \mathbb{P}[\ln z \leq \ln z_{\mathrm{A}}^-] = \alpha/2 \quad &\Rightarrow \quad \mathbb{P}[z \leq z_{\mathrm{A}}^-] = \alpha/2 \\ \mathbb{P}[\ln z \geq \ln z_{\mathrm{A}}^+] = \alpha/2 \quad &\Rightarrow \quad \mathbb{P}[z \geq z_{\mathrm{A}}^+] = \alpha/2, \end{aligned} \tag{8}$$

which provides a prediction interval for $\ln z$ and for $z$ of confidence level $1-\alpha$. To make the latter optimal, Greene (2012) proposes to minimize the length of the poverty line interval $z$, denoted by $W$.The problem thus amounts to solving a minimization program for $W$ under constraint:

$$\min_{z_{\mathrm{A}}^-, z_{\mathrm{A}}^+} W = z_{\mathrm{A}}^+ - z_{\mathrm{A}}^- \quad \text{under the constraint} \quad F(z_{\mathrm{A}}^-) + [1 - F(z_{\mathrm{A}}^+)] = \alpha, \tag{9}$$

with $F$ the distribution function of the variable $z$.

Solving the first-order conditions associated with the Lagrangian of this program leads to $f(z_{\text{A}}^{-}) = f(z_{\text{A}}^{+})$ where $f(z_{\text{A}}^{-})$ and $f(z_{\text{A}}^{+})$ denote the values of the density of $z$ at the points $z_{\text{A}}^{-}$ and $z_{\text{A}}^{+}$ respectively.[7] The solution, $CI_{A}^{z} = [z_{\text{A}}^{-}; z_{\text{A}}^{+}]$, is therefore by construction, the shortest confidence interval for $z$.

This method applies to any distribution, whether it is symmetrical or not. However, in asymmetric cases, Greene (2012) shows that it significantly improves accuracy. We apply this method because the variable $z$ is asymmetric and follows a Lognormal distribution; $\ln z$ is assumed to follow a Normal distribution. However, this method does not necessarily take into account the uncertainty relating to the estimation of the retransformation factor (smear) proposed by Duan (1983). Such uncertainty should be taken into account. In the next section, we explicitly consider it and construct new confidence intervals (Method B).

## 3.2 Biais asymptotically corrected confidence interval

The prediction corrected by the smearing factor $\hat{z}_{i,t} = \hat{h}\,\exp(x'_{i,t}\hat{\beta})$ (Equation 4), depends of two estimators: (i) the prediction $\widehat{\ln z_{i,t}} = x'_{0t}\hat{\beta}$ and (ii) Duan's smearing factor, $\hat{h}$. To construct a confidence interval for the poverty line $z_{i,t}$, we use the *multivariate delta method* (Oehlert, 1992; Wooldridge, 2010).

Duan, 1983 and Greene, 2012 show that for a sufficiently large sample size, we can assume asymptotic independence between $\hat{\beta}$ and $\hat{h}$ and thus $\text{Cov}(\widehat{\ln z_{it}}, \hat{h}) = 0$. The estimated asymptotic variance of $\hat{z}_{i,t}$ can then be written:

$$\widehat{\text{Var}}(\hat{z}_{i,t}) \;=\; \hat{z}_{i,t}^{2}\Big[x'_{i,t}\widehat{\text{Var}}(\hat{\beta})x_{i,t} \;+\; \widehat{\text{Var}}(\ln\hat{h})\Big]. \tag{10}$$

This is obtained by applying the delta method to $\hat{h}$ and to its logarithm such that:

$$\widehat{Var}(\hat{h}) \;=\; \frac{\widehat{Var}(e_{i,t})}{nT} \qquad \textbf{and} \qquad \widehat{Var}(\ln\hat{h}) \;\simeq\; \frac{\widehat{Var}(e_{i,t})}{nT\,\hat{h}^{2}}. \tag{11}$$

We can then deduce the corrected confidence interval at significance level $(1-\alpha)$ such that:

$$CI_{B}^{z}\big(\hat{z}_{i,t}\big) \;=\; \Big[\hat{z}_{i,t} \;\pm\; t_{\,nT-K,\,1-\alpha/2}\,\widehat{\text{se}}\Big], \tag{12}$$

[7] Let's recall that the density of the variable $z$ is asymmetric.

with $\widehat{\text{se}} = \sqrt{\widehat{\text{Var}}(\hat{z}_{i,t})}$ and $t_{nT-K,\,(1-\alpha)/2}$ the Student quantile with $(nT-K)$ degrees of freedom, $K$ being the number of parameters in $\beta$.[8]

By doing this, we correct the bias associated with the exponential transformation and the two uncertainties associated with the two estimated components. This gives us an alternative to Method A, which was developed previously.

On the basis of one or other of the two methods proposed so far, which takes into account the uncertainty relating to the estimation of the poverty line, we now propose to construct confidence intervals for FGT poverty measures. To do so, we use a projection method.

## 3.3 Projection approach to FGT confidence intervals

The approach we propose to capture the uncertainty around the punctual estimation of $FGT$ through the construction of confidence intervals, is inspired by Rao (1973) adopted and detailed by Abdelkhalek and Dufour (1998). It is a method based on projections of regions or confidence intervals in the scalar case.

Let $CI$ be a confidence interval of level $(1-\alpha)$, constructed by one of the methods developed in this article, that captures the uncertainty associated with estimating the threshold $z$.[9] We therefore consider that:

$$P[z \in CI] \geq 1 - \alpha \tag{13}$$

where $CI$ is a random subset of $\mathbb{R}$ with $0 \leqslant \alpha < 1$.

Clearly, for any value of the poverty line $z \in CI$ corresponds a possibly different value of each measure of type $FGT_a(z)$. Conversely, any value of $FGT_a(z)$ would correspond to at least one value of $z \in CI$. Let's denote $g(CI)$, the image set of $CI$ in $\mathbb{R}$ through the function $g(.)$ such that $g : \mathbb{R} \to \mathbb{R}$ corresponds to the function for calculating $FGT$ poverty measures given the poverty line $z$. So, in our case, $\forall z \in CI, g(z) = FGT_a(z) \in g(CI)$. As Rao (1973) shows, $g(CI)$ is a confidence interval of level $(1-\alpha)$ for $g(z)$ and therefore for $FGT_a(z)$. Indeed, since $z \in CI \Rightarrow g(z) \in g(CI)$ then

$$P[g(z) \in g(CI)] \geq P[z \in CI] \geq 1 - \alpha. \tag{14}$$

[8] The remainder of this article will note this interval $CI_B^z(\hat{z})$.

[9] Here again, to avoid overly complicated notation, we ignore the indices $i$ and $t$.

Furthermore, let $g^{-}(CI)$ and $g^{+}(CI)$ be respectively the lower and upper bounds of the confidence interval of $g(z)$ and defined such that:

$$\begin{aligned} g^{-}(CI) &:= \inf\{g(z): \quad z \in CI\} \\ g^{+}(CI) &:= \sup\{g(z): \quad z \in CI\}. \end{aligned} \tag{15}$$

Therefore, for all $z \in CI$, we deduce:

$$g(z) \in g(CI) \Rightarrow g^{-}(CI) \leq g(z) \leq g^{+}(CI) \tag{16}$$

and by combining (14) and (16):

$$P\left[g^{-}(CI) \leq g(z) \leq g^{+}(CI)\right] \;\geq\; P\left[g(z) \in g(CI)\right] \;\geq\; P[z \in CI] \;\geq\; 1-\alpha. \tag{17}$$

This means that $[g^{-}(CI); g^{+}(CI)]$ is a confidence interval of level $(1-\alpha)$ for $g(z) = FGT_a(z)$. We will denote these intervals $CI^{FGT} = [FGT^{a-}; FGT^{a+}]$. As mentioned earlier, the $FGT$ poverty measures are monotonically increasing for $z$ for a given distribution and therefore

$$CI^{FGT} = [FGT^{a-}; FGT^{a+}] = [g^{-}(CI); g^{+}(CI)] = [g(z)^{-}; g(z)^{+}]. \tag{18}$$

This projection approach to construct confidence intervals for poverty measures is used in Methods A, B and C, to be developed in the second axis (Section 4).

This second method (Method B) is by construction based on several approximations inherent in the retransformation correction and the delta method for analytically deducing the variance of the $\hat{z}_{i,t}$ prediction. The bootstrap approach proposed by Efron (1987) circumvents this problem. The two methods developed in the following section are based on this approach.

# 4 Confidence intervals by Bootstrap - BCa approach

In this section, we introduce the *Bias-Corrected and Accelerated* (BCa) method developed by Efron (1987) to construct confidence intervals adjusted for retransformation bias (Duan (1983)). This BCa approach offers a suitable non-parametric alternative to avoid asymptotic approximations for estimating the variance of the predicted poverty line. The Method C we are here developing, complements Method B by offering bootstrap-based inference.

As we have already seen, estimating the $z$ poverty line and FGT-type poverty measures (Foster et al. (1984)) should take into account the associated uncertainty. Bootstrap methods are particularly appropriate for this purpose in situations where classical asymptotic properties are difficult to justify or where the analytical calculation of the estimator variance is complex or impossible. Several methods for constructing confidence intervals using the bootstrap method have been proposed in the literature.

In this article, we apply the BCa method, introduced by Efron (1987), developed in DiCiccio and Efron (1996) and formalized by Justus et al. (2024) among others.[10] It is indeed the most appropriate when the estimator distribution is biased or asymmetric, which is often the case when the parameter of interest is obtained using nonlinear projections, as in our case. It allows us to obtain the bounds of an interval by taking into account the systematic bias of the estimator, $\hat{v}_0$. It also takes into account the speed of variation of its distribution, $\widehat{ac}_0$.[11] These adjustments are estimated using a *Jackknife* procedure.

Formally, Efron (1987) defines the bias parameter $\hat{v}_0$ as:

$$\hat{v}_0 = \Phi^{-1}\left(\frac{\#\{\hat{z}^r < \hat{z}\}}{R}\right) \tag{19}$$

where $\Phi^{-1}$ is the inverse cumulative distribution function of a centered reduced Normal distribution and $R$, the number of replicates. As before, $\hat{z}$ is the estimate of the poverty line deduced from the Model 2 for $z$ corrected by the Duan method. We denote $\hat{z}^r$ the $r^{th}$ bootstrap replication of $\hat{z}$ and $\Phi^{-1}$ represents the proportion of bootstrap replicas below $\hat{z}$.

Following Justus et al. (2024), the acceleration parameter $ac$ is estimated by the *jackknife* method as follows:

$$\widehat{ac} = \frac{1}{6}\frac{\sum_{i=1}^{n}\left(\bar{\hat{z}}_{(\cdot)} - \hat{z}_{(i)}\right)^3}{\left[\sum_{i=1}^{n}\left(\bar{\hat{z}}_{(\cdot)} - \hat{z}_{(i)}\right)^2\right]^{3/2}} \tag{20}$$

where the $\hat{z}_{(i)}$ are obtained by estimating $z$ from the Model 2 by removing the observation $i$ from the sample and $\bar{\hat{z}}_{(\cdot)}$ is the average of the $\hat{z}_{(i)}$ i.e. $\bar{\hat{z}}_{(\cdot)} = \left(\frac{1}{n}\right)\sum_{i=1}^{n}\hat{z}_{(i)}$.

[10] Other approaches exist, such as the *bootstrap-t* method or *intervals based on Bayesian approximation* (Hall (1992), Davison and Hinkley (1997)).

[11] Efron (1987) notes the bias $\hat{z}_0$. Since in this article, $z$ represents the poverty line, the bias is here noted $\hat{v}_0$. Similarly, we denote $\widehat{ac}_0$ instead of $\hat{a}_0$ for the acceleration parameter, since $a$ is the poverty aversion parameter in this article.

We then deduce the confidence interval BCa of the poverty line $z$ such that:

$$\mathrm{CI}_C^z = \left[\hat{z}^-_{(\alpha_1)}, \hat{z}^+_{(\alpha_2)}\right] \tag{21}$$

with the corrected levels defined by:

$$\alpha_1 = \Phi\left(\hat{v}_0 + \frac{\hat{v}_0 + v_{\alpha/2}}{1 - \widehat{ac}(\hat{v}_0 + v_{\alpha/2})}\right), \tag{22}$$

$$\alpha_2 = \Phi\left(\hat{v}_0 + \frac{\hat{v}_0 + v_{1-\alpha/2}}{1 - \widehat{ac}(\hat{v}_0 + v_{1-\alpha/2})}\right), \tag{23}$$

where

$$v_{\alpha/2} = \Phi^{-1}(\alpha/2) \quad \text{and} \quad v_{1-\alpha/2} = \Phi^{-1}(1-\alpha/2)$$

the quantiles of a centered reduced Normal distribution associated with the required confidence level. This interval $\mathrm{CI}_C^z$ is then projected as in Methods A and B to obtain the confidence intervals for the poverty measures, which we denote:

$$\mathrm{CI}_C^{FGT} = \left[FGT_C^{a-}, FGT_C^{a+}\right]$$

This bootstrap approach can be applied to the poverty line, as we have just discussed, but also directly to the $FGT$ poverty measures. This is what we apply in Method D, in which we estimate $R$ times the Model 2 from which we obtain $\hat{z}^r$ still considering the Duan correction. From this, we obtain $R$ estimates of the poverty measures $\widehat{FGT}_D^{ra}$ and then directly the BCa confidence intervals, denoted as follows:

$$\mathrm{CI}_D^{FGT} = \left[FGT_D^{a-}, FGT_D^{a+}\right]$$

Unlike the confidence intervals obtained with Method C, these are not based on the projection technique but are obtained from the Efron (1987) method applied directly to the poverty measures as parameters of interest in a similar way to what we developed for the $z$ poverty line.

# 5 Overview of the four methods

Four methods have been developed to construct confidence intervals for poverty lines and measures. As detailed above, these methods are based on projection or bootstrap techniques.

For the first axis, Methods A and B have a classical basis in wich we start by estimating the Model 2 to infer the prediction $\widehat{\ln z}$. In Method A, a standard confidence interval is constructed for $\ln z$, $\text{CI}_A^{lnz}$. From this, we derive a *naive* confidence interval for $z$, $\text{CI}_A^{zn}$. Since this interval is not optimal, we optimize it using the approach proposed by Greene (2012) to obtain the interval $\text{CI}_A^z$. In Method B, after estimating Model 2, the asymptotic correction proposed by Duan (1983) is introduced to obtain a new confidence interval, denoted $\text{CI}_B^z$.

For Methods C and D, a bootstrap approach is adopted in the second axis. The Model 2 is estimated $R$ times and the Duan asymptotic correction is also applied under each $r$ replicate. At the end of the process, we obtain $R$ estimates of the $z$ threshold. Under Method C, we construct the confidence interval $\text{CI}_C^z$ based on the BCa method developed by Efron (1987).

The three confidence intervals constructed for the poverty line $z$, $\text{CI}_A^z$, $\text{CI}_B^z$ and $\text{CI}_C^z$, are then projected using the Rao (1973) method to obtain the corresponding confidence intervals for the three poverty measures, $FGT^a$ i.e. $\text{CI}_A^{FGT}$, $\text{CI}_B^{FGT}$ and $\text{CI}_C^{FGT}$ for $a = 0, 1, 2$.

To obtain these intervals with Method D, for each replicate $r$, we calculate the poverty measures $FGT^{ar}$. We then apply the Bootstrap BCa approach to directly obtain $\text{CI}_D^{FGT}$ confidence intervals for $a = 0, 1, 2$. Table 1 summarizes these four methods developed in this article.

Table 1: Overview of the four methods

| **Method A** | **Method B** | **Method C** | **Method D** |
|---|---|---|---|
| **Estimation** | | | |
| **Classical** | | **Bootstrap** | |
| $\ln z = X\beta + \varepsilon$ | $\ln z = X\beta + \varepsilon$ | $\ln z^r = X\beta + \varepsilon$ | $\ln z^r = X\beta + \varepsilon$ |
| $\Downarrow$ | $\Downarrow$ | $\Downarrow$ | $\Downarrow$ |
| $\widehat{\ln z}$ | $\hat{z}$ Duan asymptotic correction | $\hat{z}$ Duan asymptotic correction bootstrapped for $r = 1, \dots, R$ | |
| $\Downarrow$ | $\Downarrow$ | $\Downarrow$ | $\Downarrow$ |
| $\mathrm{CI}_A^{lnz} = \left[ln\hat{z}_A^-, ln\hat{z}_A^+\right]$ | | | |
| $\Downarrow$ | | | |
| $\mathrm{CI}_A^{zn} = \left[\hat{zn}_A^-, \hat{zn}_A^+\right]$ | | Efron - Bias-corrected and accelerated on $z$ | |
| Greene optimized projection | | $\Downarrow$ | |
| $\mathrm{CI}_A^z = \left[\hat{z}_A^-, \hat{z}_A^+\right]$ | $\mathrm{CI}_B^z = \left[\hat{z}_B^-, \hat{z}_B^+\right]$ | $\mathrm{CI}_C^z = \left[\hat{z}_C^-, \hat{z}_C^+\right]$ | |
| Projection on $FGT^a$ with $a = 0, 1, 2$ | | | Efron - Bias-corrected and accelerated on $FGT$ |
| $\mathrm{CI}_A^{FGT} = \left[FGT_A^{a-}, FGT_A^{a+}\right]$ | $\mathrm{CI}_B^{FGT} = \left[FGT_B^{a-}, FGT_B^{a+}\right]$ | $\mathrm{CI}_C^{FGT} = \left[FGT_C^{a-}, FGT_C^{a+}\right]$ | $\mathrm{CI}_D^{FGT} = \left[FGT_D^{a-}, FGT_D^{a+}\right]$ |

# 6 Illustration for the Moroccan case

To illustrate these approaches, the estimation of the Model 2 is performed on 1,049 country-year observations covering 151 countries between 1963 and 2024, drawn from *PovcalNet* and *Global Monitoring Database* (World Bank).[12]

For the Moroccan application, the surveys used in this article are the *Enquêtes nationales sur la consommation et les dépenses des ménages* (ENCDM) of 2000-01, 2014, the *Enquête nationale sur le niveau de vie des ménages* (ENNVM) of 2006-07. We also use data from waves 2012 and 2019 of the *Enquête Panel de ménages* (EPM) of the *Observatoire National du Développement Humain* (ONDH).[13] These five surveys provide representative results at national level, as well as in urban and rural areas. As the thresholds are different for each area of residence, the ten results are derived separately.

We use the results of the estimation of Model 2, reported in Table 11 in the Appendix, to generate the predicted poverty lines $\hat{z}$. The results are consistent with those produced by ESCWA (2022) in terms of the signs and amplitudes of the estimates. The goodness of fit is excellent (Ajusted $R^2 = 0.8532$). The scatter plot and fitted values plot (Figure 1) confirms the linear relationship between $\ln z$ and $\ln M$, as supported by ESCWA (2022) on the basis of Engel's law.

Figures 2a and 2b clearly illustrate that the density of the $z$ threshold cannot be considered as a Normal, unlike that of $\ln z$ and therefore of $\widehat{\ln z}$. These two observations justify the use of the Model 2 to make predictions and construct confidence intervals based on the four proposed methods. All the intervals constructed are of level 95% i.e $\alpha = 5\%$.

Table 2 reports the confidence intervals of $z$ obtained by the method termed naive by Greene (2012) and those optimized (Method A).[14] As expected after optimization, the latter are all shorter, while remaining fairly wide, reflecting the uncertainty involved in predicting a poverty line and the resulting measures (Table 3).

In Table 4, we compare the punctual values of the observed and predicted thresh-

[12] We thank the ESCWA team for making available all the database built and used in ESCWA (2022) and updated since.

[13] The ENNVM of 2022 and the EPM of 2023 are not available, but the work could very easily be updated.

[14] To obtain these intervals, we numerically solved the program 9 for the 1,049 observations in the ESCWA (2022) base sample (this was not necessary) and for the ten additional observations relating to the Moroccan case, using GAMS software.

Table 2: Comparison of naive and Greene-optimized CIs - Method A

| YearGroup | CI $z_{naive}$ | CI $z_A$ | Gap |
|---|---|---|---|
| 2001 Urban | [1375.43 ; 3839.61] | [1263.47 ; 3645.23] | -82.42 |
| 2001 Rural | [866.29 ; 2409.13] | [796.32 ; 2287.91] | -51.25 |
| 2007 Urban | [1712.72 ; 4785.94] | [1573.02 ; 4543.27] | -102.97 |
| 2007 Rural | [1155.51 ; 3217.57] | [1061.93 ; 3055.33] | -68.66 |
| 2012 Urban | [2287.37 ; 6405.82] | [2099.95 ; 6079.87] | -138.53 |
| 2012 Rural | [1361.33 ; 3793.37] | [1250.93 ; 3601.89] | -81.08 |
| 2014 Urban | [2231.01 ; 6244.75] | [2048.40 ; 5927.26] | -134.88 |
| 2014 Rural | [1466.78 ; 4088.97] | [1347.71 ; 3882.42] | -87.49 |
| 2019 Urban | [2513.25 ; 7039.26] | [2307.27 ; 6681.00] | -152.27 |
| 2019 Rural | [1638.90 ; 4570.85] | [1505.74 ; 4339.79] | -97.90 |

Sources: Data from HCP, ONDH and authors' calculations.
Note: Values are in MAD per year and per person.

Table 3: CI of FGT indices (projection from Greene $CI_{Z_A}$ ) - Method A

| YearGroup | CI $FGT0_A$ | CI $FGT1_A$ | CI $FGT2_A$ |
|---|---|---|---|
| 2001 Urban | [0.0007 ; 0.0968] | [0.0001 ; 0.0195] | [0.0000 ; 0.0062] |
| 2001 Rural | [0.0021 ; 0.0949] | [0.0004 ; 0.0208] | [0.0001 ; 0.0070] |
| 2007 Urban | [0.0000 ; 0.0845] | [0.0000 ; 0.0170] | [0.0000 ; 0.0051] |
| 2007 Rural | [0.0000 ; 0.0925] | [0.0000 ; 0.0200] | [0.0000 ; 0.0066] |
| 2012 Urban | [0.0025 ; 0.0797] | [0.0003 ; 0.0185] | [0.0001 ; 0.0067] |
| 2012 Rural | [0.0004 ; 0.0790] | [0.0002 ; 0.0138] | [0.0001 ; 0.0037] |
| 2014 Urban | [0.0000 ; 0.0506] | [0.0000 ; 0.0092] | [0.0000 ; 0.0028] |
| 2014 Rural | [0.0006 ; 0.0617] | [0.0001 ; 0.0119] | [0.0000 ; 0.0037] |
| 2019 Urban | [0.0000 ; 0.0129] | [0.0000 ; 0.0018] | [0.0000 ; 0.0004] |
| 2019 Rural | [0.0000 ; 0.0194] | [0.0000 ; 0.0020] | [0.0000 ; 0.0003] |

olds after correction by the Duan approach and the associated poverty incidence measures ($FGT0$). We find significant discrepancies between the results thus obtained, particularly in rural areas. Despite the quality of the adjustment, we obtain significant differences between predicted and observed values, which are not solely due to the non-correction based on Duan's *smear* coefficient. In our case, the estimate of this coefficient is only 1.0336. It is interesting to note that with this coefficient, the predicted poverty lines are all increased by 3.36%. This has a significant impact on poverty measures, particularly for countries and years for which the predicted value $\hat{z}$ is at a level where the density of the variable of interest is high. This means that for some countries, the CPL approach leaves clearly room for a great deal of uncertainty, which needs to be translated into confidence intervals.

Table 4: Comparison of observed, naive and Duan punctual values: $z$ and $FGT0$

| YearGroup | z_Obs | z_Naive | z_Duan | FGT0_Obs | FGT0_Naive | FGT0_Duan |
|---|---|---|---|---|---|---|
| 2001 Urban | 3421.00 | 2298.07 | 2375.36 | 0.0767 | 0.0151 | 0.0179 |
| 2001 Rural | 3098.00 | 1444.65 | 1493.24 | 0.2514 | 0.0171 | 0.0186 |
| 2007 Urban | 3834.00 | 2863.04 | 2959.34 | 0.0475 | 0.0124 | 0.0149 |
| 2007 Rural | 3569.00 | 1928.19 | 1993.05 | 0.1475 | 0.0168 | 0.0198 |
| 2012 Urban | 4423.00 | 3827.86 | 3956.60 | 0.0259 | 0.0163 | 0.0176 |
| 2012 Rural | 4117.00 | 2272.45 | 2348.89 | 0.1238 | 0.0027 | 0.0074 |
| 2014 Urban | 4667.00 | 3732.58 | 3858.12 | 0.0161 | 0.0063 | 0.0070 |
| 2014 Rural | 4312.00 | 2449.00 | 2531.37 | 0.0932 | 0.0087 | 0.0100 |
| 2019 Urban | 4939.40 | 4206.12 | 4347.59 | 0.0012 | 0.0002 | 0.0002 |
| 2019 Rural | 4563.40 | 2737.00 | 2829.06 | 0.0258 | 0.0001 | 0.0001 |

Sources : Data from HCP, ONDH and authors' calculations.
Notes : Values of $z$ are in MAD per year and per person.
$z_{\text{Obs}}$ is a relative poverty line—computed on the five surveys and by area of residence—as 50% of median household *per capita* expenditure. The observed poverty incidence $FGT0_{Obs}$ is derived from these poverty lines.

Table 5 precisely reports the confidence intervals of $z$ noted $CIz_{DuanB}$ and those of the associated poverty measures obtained by projection (Method B). Naturally, all point values in Table 4 relating to $z_{Duan}$ and $FGT0_{Duan}$ are covered by the corresponding confidence intervals. It is notable, however, that the intervals constructed by Method B remain wider than those obtained with Method A after optimization (see Table 7).[15]

Table 5: Confidence intervals for $z_{Duan}$ and FGT measures (Proj) - Method B

| YearGroup | CI zDuan$_B$ | CI FGT0$_B$ | CI FGT1$_B$ | CI FGT2$_B$ |
|---|---|---|---|---|
| 2001 Urban | [1156.08 ; 3594.64] | [0.0005 ; 0.0911] | [0.0001 ; 0.0185] | [0.0000 ; 0.0058] |
| 2001 Rural | [729.60 ; 2256.88] | [0.0013 ; 0.0919] | [0.0003 ; 0.0198] | [0.0001 ; 0.0067] |
| 2007 Urban | [1438.83 ; 4479.84] | [0.0000 ; 0.0795] | [0.0000 ; 0.0160] | [0.0000 ; 0.0048] |
| 2007 Rural | [972.52 ; 3013.57] | [0.0004 ; 0.0902] | [0.0001 ; 0.0191] | [0.0000 ; 0.0063] |
| 2012 Urban | [1919.34 ; 5993.87] | [0.0013 ; 0.0763] | [0.0002 ; 0.0176] | [0.0000 ; 0.0063] |
| 2012 Rural | [1145.33 ; 3552.45] | [0.0004 ; 0.0733] | [0.0001 ; 0.0130] | [0.0001 ; 0.0035] |
| 2014 Urban | [1872.57 ; 5843.67] | [0.0000 ; 0.0479] | [0.0000 ; 0.0086] | [0.0000 ; 0.0026] |
| 2014 Rural | [1233.76 ; 3828.99] | [0.0006 ; 0.0574] | [0.0000 ; 0.0112] | [0.0000 ; 0.0034] |
| 2019 Urban | [2108.74 ; 6586.43] | [0.0000 ; 0.0122] | [0.0000 ; 0.0016] | [0.0000 ; 0.0003] |
| 2019 Rural | [1378.21 ; 4279.90] | [0.0000 ; 0.0178] | [0.0000 ; 0.0017] | [0.0000 ; 0.0003] |

Sources: Data from HCP, ONDH and authors' calculations.
Note: Values of $z$ are in MAD per year and per person.

[15] Table 7 reports the widths of the poverty line confidence intervals constructed by the three methods.

As introduced in the previous section, Method C combines a BCa bootstrap approach to construct confidence intervals for the threshold $z$ and the projection method for the confidence intervals of the monetary poverty measures considered. These confidence intervals are shown in Table 6. Those for $z$ cover the point values predicted using the Duan correction reported in Table 4. It is the bounds of these intervals that generate, by projection, those of the poverty measures presented in Table 6. Table 7 show that Method C produces shorter intervals than those obtained by Methods A and B whatever the year and the aera.

The logical outcome of the four developped methods is to construct Bootstrap BCa confidence intervals for the poverty measures without using the projection method. Table 8 reports these intervals.[16] These intervals are clearly the shortest. As illustrated in Table 1, the poverty lines are estimated by applying the asymmetric correction proposed by Duan (1983).

Table 6: BCa confidence intervals for $z_C$ and FGT measures (Proj) - Method C

| YearGroup | CI zBCa$_C$ | CI FGT0$_C$ | CI FGT1$_C$ | CI FGT2$_C$ |
|---|---|---|---|---|
| 2001 Urban | [2025.23 ; 2585.14] | [0.0084 ; 0.0250] | [0.0012 ; 0.0044] | [0.0003 ; 0.0012] |
| 2001 Rural | [1312.92 ; 1583.39] | [0.0113 ; 0.0259] | [0.0021 ; 0.0047] | [0.0008 ; 0.0016] |
| 2007 Urban | [2505.88 ; 3239.43] | [0.0043 ; 0.0202] | [0.0007 ; 0.0034] | [0.0002 ; 0.0009] |
| 2007 Rural | [1732.46 ; 2133.88] | [0.0112 ; 0.0266] | [0.0017 ; 0.0047] | [0.0004 ; 0.0013] |
| 2012 Urban | [3303.45 ; 4382.98] | [0.0089 ; 0.0257] | [0.0020 ; 0.0057] | [0.0007 ; 0.0020] |
| 2012 Rural | [2030.12 ; 2527.08] | [0.0021 ; 0.0162] | [0.0004 ; 0.0015] | [0.0002 ; 0.0004] |
| 2014 Urban | [3231.55 ; 4262.34] | [0.0032 ; 0.0113] | [0.0006 ; 0.0021] | [0.0001 ; 0.0006] |
| 2014 Rural | [2180.34 ; 2731.32] | [0.0042 ; 0.0137] | [0.0008 ; 0.0024] | [0.0002 ; 0.0007] |
| 2019 Urban | [3627.23 ; 4819.09] | [0.0000 ; 0.0011] | [0.0000 ; 0.0001] | [0.0000 ; 0.0000] |
| 2019 Rural | [2428.51 ; 3061.27] | [0.0000 ; 0.0007] | [0.0000 ; 0.0001] | [0.0000 ; 0.0000] |

Sources: Data from HCP, ONDH and authors' calculations.
Note: Values of $z$ are in MAD per year and per person.

Clearly, a reading of the results obtained with the four methods of constructing confidence intervals also supports an inferential reading in terms of hypothesis testing on both poverty lines and associated measures. These methods can be used in the same contexts as those proposed by Kakwani (1993) and Bishop et al. (1995), with the added consideration of uncertainty about poverty lines. Indeed, they develop an inference framework for decomposable monetary poverty indices (FGT class and related) and assume that the poverty line is fixed and known without uncertainty - an assumption they explicitly question. It is exactly this uncertainty that we approach in this article. We believe that our Methods can

[16] It is trivial that the confidence intervals for the associated poverty lines are identical to those obtained in Method C.

cover the gap in time (different periods) and space (between countries, regions, strata, ...) or both in terms of comparison.

Table 7: Width of confidence intervals for $z$ - Methods A, B and C

| YearGroup | $\text{Naive}_A$ | $\text{Greene}_A$ | $\text{Duan}_B$ | $\text{BCa}_C$ |
|---|---|---|---|---|
| 2001 Urban | 2464.18 | 2381.76 | 2438.56 | 559.91 |
| 2001 Rural | 1542.84 | 1491.59 | 1527.28 | 270.47 |
| 2007 Urban | 3073.22 | 2970.25 | 3041.01 | 733.55 |
| 2007 Rural | 2062.06 | 1993.4 | 2041.05 | 401.42 |
| 2012 Urban | 4118.45 | 3979.92 | 4074.53 | 1079.53 |
| 2012 Rural | 2432.04 | 2350.96 | 2407.12 | 496.96 |
| 2014 Urban | 4013.74 | 3878.86 | 3971.1 | 1030.79 |
| 2014 Rural | 2622.19 | 2534.71 | 2595.23 | 550.98 |
| 2019 Urban | 4526.01 | 4373.73 | 4477.69 | 1191.86 |
| 2019 Rural | 2931.95 | 2834.05 | 2901.69 | 632.76 |

Sources: Data from HCP, ONDH and authors' calculations.
Note: Values are in MAD per year and per person.

Table 8: CI Bootstrap BCa for FGT measures - Method D

| YearGroup | FGT0 $\text{BCa}_D$ | FGT1 $\text{BCa}_D$ | FGT2 $\text{BCa}_D$ |
|---|---|---|---|
| 2001 Urban | [0.0079 ; 0.0272] | [0.0011 ; 0.0048] | [0.0003 ; 0.0013] |
| 2001 Rural | [0.0111 ; 0.0275] | [0.0021 ; 0.0050] | [0.0008 ; 0.0017] |
| 2007 Urban | [0.0043 ; 0.0227] | [0.0007 ; 0.0038] | [0.0002 ; 0.0010] |
| 2007 Rural | [0.0112 ; 0.0273] | [0.0016 ; 0.0051] | [0.0004 ; 0.0014] |
| 2012 Urban | [0.0089 ; 0.0283] | [0.0020 ; 0.0062] | [0.0007 ; 0.0022] |
| 2012 Rural | [0.0017 ; 0.0172] | [0.0004 ; 0.0017] | [0.0002 ; 0.0004] |
| 2014 Urban | [0.0032 ; 0.0126] | [0.0005 ; 0.0023] | [0.0001 ; 0.0007] |
| 2014 Rural | [0.0041 ; 0.0146] | [0.0008 ; 0.0026] | [0.0002 ; 0.0007] |
| 2019 Urban | [0.0000 ; 0.0012] | [0.0000 ; 0.0001] | [0.0000 ; 0.0000] |
| 2019 Rural | [0.0000 ; 0.0008] | [0.0000 ; 0.0001] | [0.0000 ; 0.0000] |

Sources: Data from HCP, ONDH and authors' calculations.

Tables 9 and 10 summarize the confidence intervals constructed with the four methods developed, respectively for poverty lines and for the incidence of poverty in Morocco between 2000 and 2019 in urban and rural areas. In a complementary perspective, these same confidence intervals form the basis for conducting $\alpha = 5\%$ hypothesis tests on poverty thresholds or measures, thereby allowing one to determine whether the observed variations in these measures over time or across space are statistically significant.

Confidence intervals obtained with Method D, which are shorter than those derived from Methods A, B, and C, are liberal: for a given significance level $\alpha$,

Table 9: Confidence intervals for $z$ (CIs $z$)

| YearGroup | Methods | | |
|---|---|---|---|
| | A | B | C |
| 2001 Urban | [1263.47 ; 3645.23] | [1156.08 ; 3594.64] | [2025.23 ; 2585.14] |
| 2001 Rural | [796.32 ; 2287.91] | [729.60 ; 2256.88] | [1312.92 ; 1583.39] |
| 2007 Urban | [1573.02 ; 4543.27] | [1438.83 ; 4479.84] | [2505.88 ; 3239.43] |
| 2007 Rural | [1061.93 ; 3055.33] | [972.52 ; 3013.57] | [1732.46 ; 2133.88] |
| 2012 Urban | [2099.95 ; 6079.87] | [1919.34 ; 5993.87] | [3303.45 ; 4382.98] |
| 2012 Rural | [1250.93 ; 3601.89] | [1145.33 ; 3552.45] | [2030.12 ; 2527.08] |
| 2014 Urban | [2048.40 ; 5927.26] | [1872.57 ; 5843.67] | [3231.55 ; 4262.34] |
| 2014 Rural | [1347.71 ; 3882.42] | [1233.76 ; 3828.99] | [2180.34 ; 2731.32] |
| 2019 Urban | [2307.27 ; 6681.00] | [2108.74 ; 6586.43] | [3627.23 ; 4819.09] |
| 2019 Rural | [1505.74 ; 4339.79] | [1378.21 ; 4279.90] | [2428.51 ; 3061.27] |

Sources: Data from HCP, ONDH and authors' calculations.
Note: Values of $z$ are in MAD per year and per person.

Table 10: Confidence intervals for incidence of poverty $FGT0$ (CI $FGT0$)

| YearGroup | Methods | | | |
|---|---|---|---|---|
| | A | B | C | D |
| 2001 Urban | [0.0007 ; 0.0968] | [0.0005 ; 0.0911] | [0.0084 ; 0.0250] | [0.0079 ; 0.0272] |
| 2001 Rural | [0.0021 ; 0.0949] | [0.0013 ; 0.0919] | [0.0113 ; 0.0259] | [0.0111 ; 0.0275] |
| 2007 Urban | [0.0000 ; 0.0845] | [0.0000 ; 0.0795] | [0.0043 ; 0.0202] | [0.0043 ; 0.0227] |
| 2007 Rural | [0.0000 ; 0.0925] | [0.0004 ; 0.0902] | [0.0112 ; 0.0266] | [0.0112 ; 0.0273] |
| 2012 Urban | [0.0025 ; 0.0797] | [0.0013 ; 0.0763] | [0.0089 ; 0.0257] | [0.0089 ; 0.0283] |
| 2012 Rural | [0.0004 ; 0.0790] | [0.0004 ; 0.0733] | [0.0021 ; 0.0162] | [0.0017 ; 0.0172] |
| 2014 Urban | [0.0000 ; 0.0506] | [0.0000 ; 0.0479] | [0.0032 ; 0.0113] | [0.0032 ; 0.0126] |
| 2014 Rural | [0.0006 ; 0.0617] | [0.0006 ; 0.0574] | [0.0042 ; 0.0137] | [0.0041 ; 0.0146] |
| 2019 Urban | [0.0000 ; 0.0129] | [0.0000 ; 0.0122] | [0.0000 ; 0.0011] | [0.0000 ; 0.0012] |
| 2019 Rural | [0.0000 ; 0.0194] | [0.0000 ; 0.0178] | [0.0000 ; 0.0007] | [0.0000 ; 0.0008] |

Sources: Data from HCP, ONDH and authors' calculations.

they raise the probability of rejecting the null hypothesis of equality of poverty measures across time or space. This property is particularly relevant for assessing the effectiveness of economic policies between two surveys or across regions, since wider intervals (Methods A, B, and C) may fail to reject the null hypothesis of equality.

As an illustration, for Morocco's urban area in 2001, testing whether the poverty headcount ratio equals 7% ($H0$) leads to non-rejection with Methods A and B, whereas Method D clearly rejects the null hypothesis at the 5% level.

Furthermore, note that all that has been developed in the case of Morocco for five surveys (years) and areas of residence can be applied to construct confidence intervals and testing hypothesis for poverty measures for international comparisons in time and space. Employing these methods can strengthen the robustness of findings in contexts prone to debate, as exemplified by the report ESCWA (2023b), whose conclusions have been challenged by several of the countries concerned.

# 7 Conclusion

This article contributes to the literature on monetary poverty measurement by highlighting a rarely addressed issue: the uncertainty surrounding both the estimation of poverty lines and the poverty indicators derived from them. Building on ESCWA (2022) recent work on Concave Poverty Lines (CPL), we show that assuming a perfectly known poverty threshold is contestable, particularly when poverty is compared across time or space.

Methodologically, we develop four methods for constructing confidence intervals (CIs) for both poverty thresholds and the Foster–Greer–Thorbecke (FGT) indices. More precisely, Method A estimates the poverty line with a log-linear regression and naive exponential retransformation, then applies Greene's optimisation algorithm to shorten the interval. Method B follows the same structure but corrects the retransformation bias with Duan's smearing estimator and the usual asymptotic approach. To avoid variance estimation and asymptotic assumptions altogether, Method C constructs a BCa bootstrap CI for the poverty line, which is then projected to obtain CIs for the poverty measures. Method D eliminates the projection step by generating, *via* the BCa bootstrap, CIs directly for the monetary poverty measures themselves; this fully bootstrap-based procedure yields the shortest—and hence more liberal—intervals, increasing the power to detect temporal or spatial changes in poverty.

We apply these methods first to ESCWA's CPL data set and then to microdata from five Moroccan household surveys (2001–2019). The findings show, first, that uncertainty in estimating poverty lines is non-negligible and, second, that this uncertainty clearly affects poverty measures. The CIs produced by Method D are systematically shorter, making them liberal and raising the likelihood of rejecting equality of poverty measures across periods, countries or regions. Such comparisons and tests are crucial for evaluating public-policy effectiveness and for international assessments: excessively wide intervals can obscure genuine progress and accept equality when liberal calibration reveals statistically significant changes and differences.

Any comparison of poverty measures over time or space should therefore incorporate threshold uncertainty and its implications, otherwise international or interregional comparisons risk being biased. Future research could extend these methods to other monetary poverty measures (e.g., Sen, Watts) or to multidimensional indices (e.g., Alkire–Foster), and develop inferential techniques that explicitly account for measurement error in key variables such as expenditure or income.

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# 8 Annex

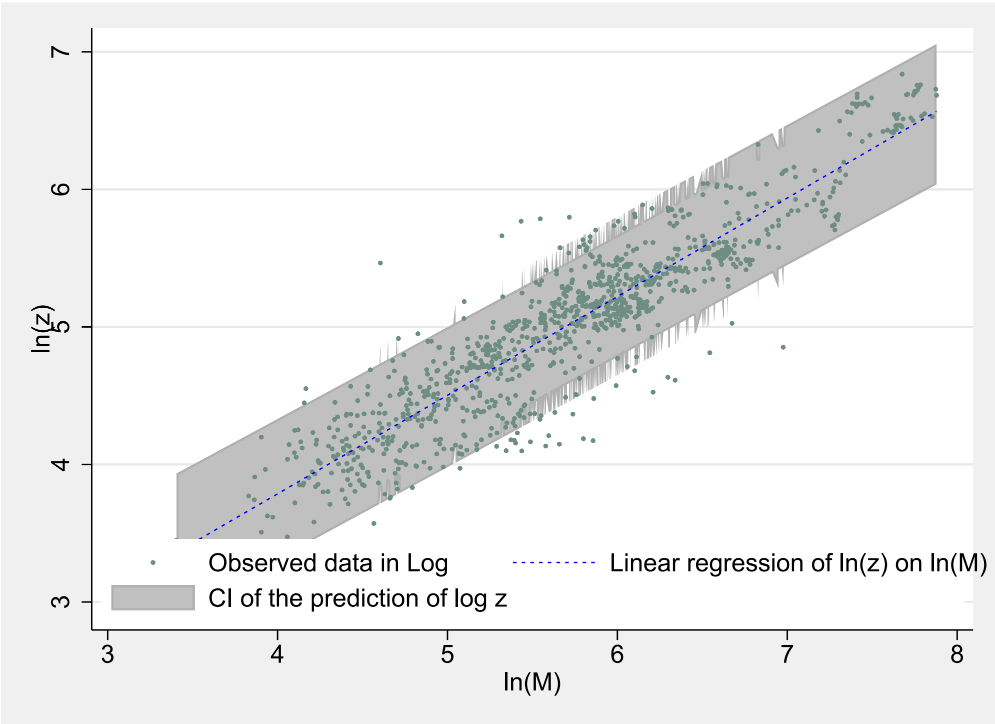
## 8.1 Annex A

Table 11: Linear regression of ln($z$)

| | Dependent variable: ln($z$) | | |
|---|---|---|---|
| | ln($M$) | $C$ | Constant |
| Coefficient | 0.6684 | 0.1334 | 1.1474 |
| $t$-value | (54.73) | (6.07) | (17.59) |
| Observations | 1,049 | | |
| F(2, 1046) | 3,045.48 | | |
| $p$-value (F-test) | 0.0000 | | |
| Adjusted $R^2$ | 0.8532 | | |

Source: ESCWA (2022) and authors' calculations.

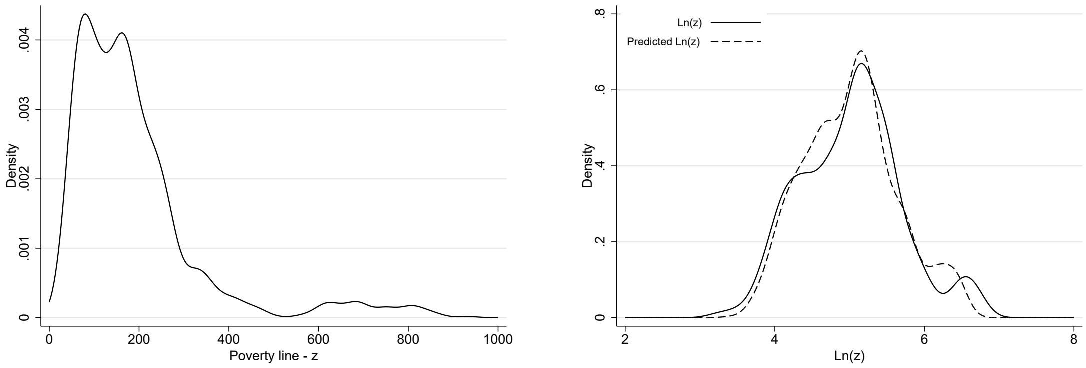
Figure 1: Linear regression of ln($z$) on ln($M$)




Source: ESCWA (2022) and authors' calculations.

Figure 2: Kernel density of poverty lines (Level and Log)

(a) Density of observed poverty line (b) Log-density: observed and predicted




Source: ESCWA (2022) and authors' calculations.

Figure 3: Comparison of confidence intervals ($z$) — 2019, Urban area